Calculus AB

4-4

(Day 2)

Mean Value Theorem

and

Average

Reviewing concepts: Mean Value Theorem for Derivatives

What are the two essential requirements for the Mean Value Theorem?

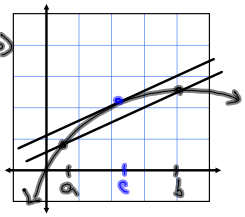
1) continuous on (a,b)

2) differentiable on $[a,b]$

What does the Mean Value Theorem state?

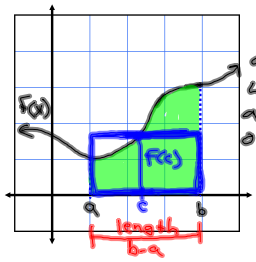
if $F(x)$ is continuous on (a,b) and differentiable on $[a,b]$ then there exists $c \in (a,b)$ such that $F(c) = \frac{F(b)-F(a)}{b-a}$

Graphically show the Mean Value Theorem.



Mean Value Theorem for Integrals

If f is continuous on the closed interval [a,b], then there exists a number $c \in [a,b]$ such that $\int_a^b f(x)\,dx = f(c)(b-a)$.



There must be a rectangle with the same area as the original shape

Find the value(s) of c guaranteed by the Mean Value Theorem for Integrals for the function over the indicated interval? (pg 293)

50) $f(x) = \cos x$ on $[-\frac{\pi}{3}, \frac{\pi}{3}]$.

$\int_{-\pi/3}^{\pi/3} \cos x\,dx = \sin x \Big|_{-\pi/3}^{\pi/3} = \sin\frac{\pi}{3} - \sin\left(-\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) = \sqrt{3}$

$\sqrt{3} = F(c)(b-a)$

$\sqrt{3} = F(c)\left(\frac{\pi}{3} - \left(-\frac{\pi}{3}\right)\right)$

$\sqrt{3} = F(c)\frac{2\pi}{3}$

$\frac{3\sqrt{3}}{2\pi} = F(c)$

$.827 = \cos c$

$\cos^{-1}(.827) = c$

$.597 = c$

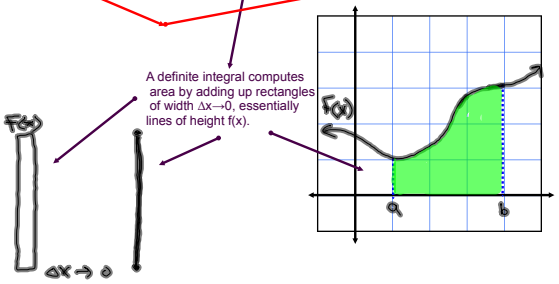
Definition of the Average Value of a Function on an interval.

If f is integrable on [a,b], then the average value of f on [a,b] is

$\frac{1}{b-a}\int_a^b f(x)\,dx$ (all the x values in [a,b]; height for each value of $x \in [a,b]$)

average = $\frac{\text{sum of all } f(x)}{\text{the number of } x \text{ values in } [a,b]}$

A definite integral computes area by adding up rectangles of width $\Delta x \to 0$, essentially lines of height f(x).



Find the average value of the function over the indicated interval and all values of x in the interval for which the function equals its average value.

52) $f(x) = \frac{4(x^2+1)}{x^2}$ [1, 3]

$\frac{1}{3-1}\int_1^3 \frac{4(x^2+1)}{x^2}\,dx = \frac{4}{2}\int_1^3 (1+x^{-2})\,dx$

$\frac{x^2}{x^2} + \frac{1}{x^2}$

$2\left[x - x^{-1}\right]_1^3 = 2\left[\left(3 - 3^{-1}\right) - \left(1 - 1^{-1}\right)\right]$

$= 2\left[\frac{8}{3} - 0\right]$

$= \frac{16}{3}$

b) $\frac{16}{3} = \frac{4(x^2+1)}{x^2} \to 4x^2 = 3x^2 + 3$

$x^2 = 3$

$x = \pm\sqrt{3}$

$\sqrt{3}$ is in $[1,3]$

61) The force F (in newtons) of a hydraulic cylinder in a press is proportional to the square of $sec(x)$, where x is the distance (in meters) that the cylinder is extended in its cycle. The domain of F is $[\ 0, \frac{\pi}{3}\ ]$, and $F(0) = 500$.

a) Find F as a function of x.

use this to find k

$F = k\sec^2 x$ $\quad 500 = k\sec^2(0)$ $\quad 500 = 1k$

$F = 500\sec^2 x$

translate: proportional means equal times a constant, k, known as the constant of proportionality or constant of variation.

b) Find the average force exerted by the press over the given interval.

$$\frac{1}{\frac{\pi}{3}-0}\int_0^{\frac{\pi}{3}} 500\sec^2 x\,dx = \frac{500}{\frac{\pi}{3}}\left[\tan x\right]_0^{\frac{\pi}{3}}$$

$$= \frac{1500}{\pi}\left[\sqrt{3}-0\right] = \frac{1500\sqrt{3}}{\pi}\ n$$

Assignment:

Pg. 285

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